CALCULATING THE HEAT RELEASE FROM FINNED AND POROUS SURFACES IN THE CASE OF A PARABOLICALLY DISTRIBUTED HEAT FLOW

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Numerical integration is used to determine the temperature fields in a surface being cooled. The working graphs are given.

Existing methods of designing extended finned surfaces make no provision for the conduction of heat from the fins in the direction in which the cooling fluid is moving, nor is any provision made for the heating of the fluid along the length of the fins, nor for the change in the heat-transfer conditions resulting from the nonuniform heating of the fluid over the height of the fins. These factors cannot be neglected when the heat flows are large and when the cooling fluid is moving in a laminar regime.

These factors have been investigated by numerous researchers in recent times in connection with the design of heat-exchangers, where the temperature distribution is specified over the length of the extended surface [1-4].

In [5, 6] we find a solution for the problem of determining the temperature field in a fin and in a fluid for the case of a constant heat flow over the length of the extended surface.

In practical problems we frequently encounter cases in which the heat flow to the cooling surface of the installation is not constant over the length of the entire surface.

Below we solve the problem of cooling surfaces when the heat flow is distributed parabolically over the surface length, since this most closely corresponds to the true distribution of the heat flow in devices that are being cooled.

The problem is described by a system of differential equations: Eq. (1) for the transfer of heat in the fin or in a porous strip that is being cooled and Eq. (5) for the heat balance of the cooling fluid, with the corresponding boundary conditions (2)-(4) and (6), which are presented below in dimensionless form (additional information, in greater detail, as to the formulation of the problem can be found in [5]):

$$\bar{\lambda}\left(\frac{\partial^2\theta}{\partial\xi^2}+\frac{\partial^2\theta}{\partial\eta^2}\right)=\varepsilon(\theta-\varphi), \tag{1}$$

$$\xi=0;\ \xi=B_0;\ \frac{\partial\theta}{\partial\xi}=0, \tag{2}$$

$$\eta=1;\ \frac{\partial\theta}{\partial\eta}=0, \tag{3}$$

$$\eta=0;\ \frac{\partial\theta}{\partial\eta}=-6(\bar{\xi}-\bar{\xi}^2), \tag{4}$$

$$\beta\frac{\partial\varphi}{\partial\xi}=\theta-\varphi, \tag{5}$$

$$\xi=0;\ \varphi=0. \tag{6}$$

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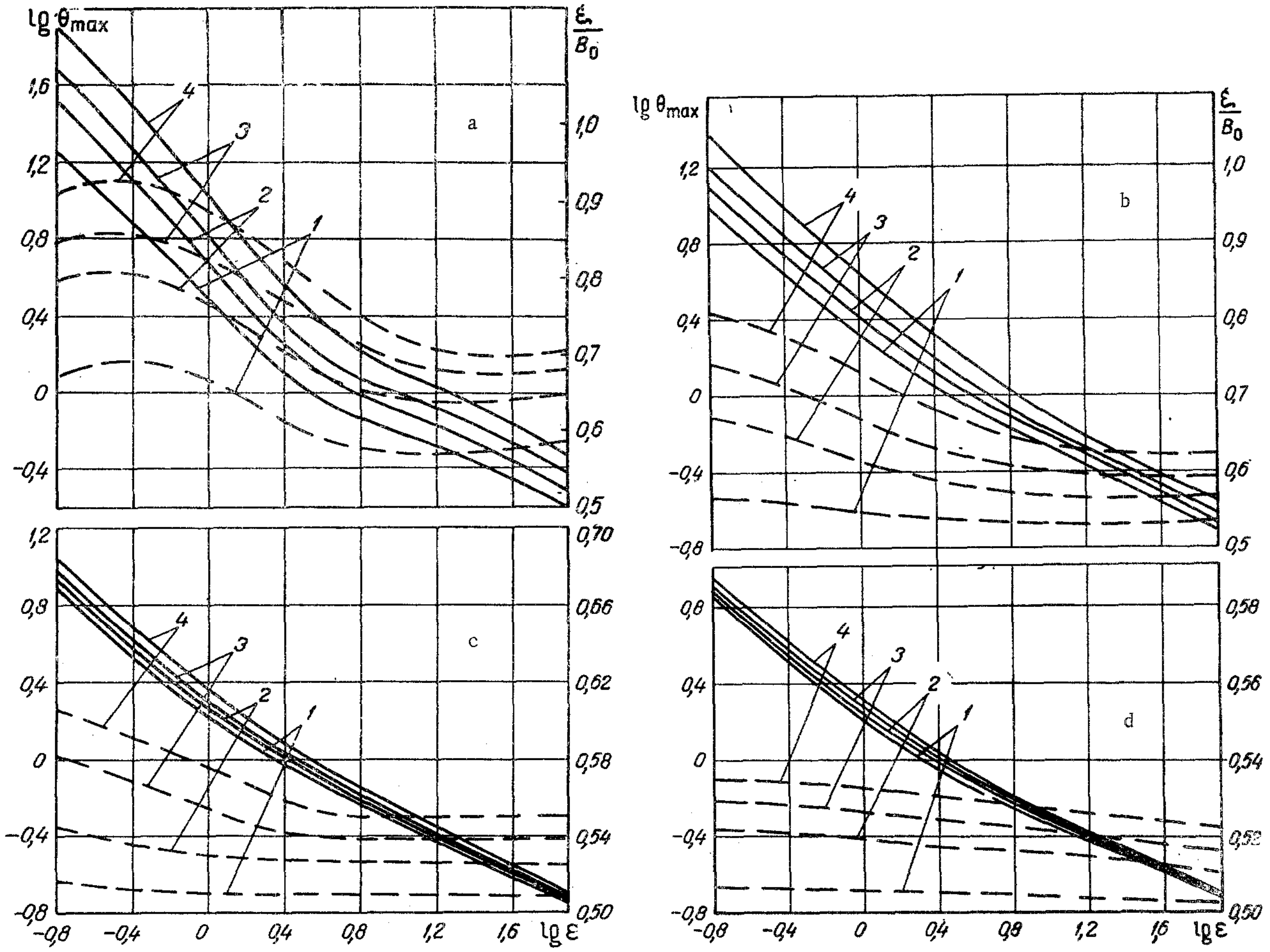

Fig. 1. The functions $\theta_{max} = f(\varepsilon, \beta, B_0)$ (solid lines) and $\xi/B_0 = f(\varepsilon, \beta, B_0)$ (dashed lines) for $\beta = 0.8$ (a); 3.2 (b); 12.8 (c); and 51.2 (d): 1) $B_0 = 2$; 2) 4; 3) 6; 4) 10.

Here

$$\xi = \frac{x}{h}; \quad \eta = \frac{y}{h}; \quad B_0 = \frac{B}{h}; \quad \bar{\xi} = \frac{\xi}{B_0};$$

a) for a finned surface

$$\varepsilon = \frac{2\alpha h^2}{\lambda\delta}; \quad \beta = \frac{cv\varkappa\gamma}{2\alpha h}; \quad \theta = \frac{\lambda\delta}{q_{av}(\delta + \varkappa) h} t;$$

$$\varphi = \frac{\lambda\delta}{q_{av}(\delta + \varkappa) h} u; \quad \bar{\lambda} = 1;$$

b) for a porous cooled strip

$$\bar{\lambda} = \frac{\lambda_{eq1}}{\lambda_{eq2}} = 1; \quad \varepsilon = \frac{\alpha p h^2}{\lambda_{eq2}}; \quad \beta = \frac{cG}{\alpha p h};$$

$$\theta = \frac{\lambda_{eq2}}{q_{av} h} t; \quad \varphi = \frac{\lambda_{eq2}}{q_{av} h} u.$$

The system of equations (1)-(6) was solved by means of finite integral transformations. We derived expressions in the form of infinite series, which enabled us to determine the temperatures for the fluid and for the fins, or that of the porous material.

To find the working formulas which can be applied to practical problems, the authors employed the method of fractional intervals for the numerical integration of the system of equations (1)-(6).

We adopted the Douglas–Rachford stabilizing correction-factor regime [7] as the iteration scheme for the solution of problems (1)-(6), with the fluid temperature regarded as constant in this case for each integration interval. The system of equations (1)-(6), in fractional intervals, can be presented in the following form:

$$\frac{\theta_{i,j}^{n+\frac{1}{2}}-\theta_{i,j}^{n}}{\tau}=\Lambda_1\theta_{i,j}^{n+\frac{1}{2}}+\Lambda_2\theta_{i,j}^{n}-\varepsilon\left(\theta_{i,j}^{n}-\varphi_{i,j}^{n}\right), \tag{7}$$

$$i=0;\ i=m-1;\ \frac{\theta_{i+1,j}^{n+\frac{1}{2}}-\theta_{i,j}^{n+\frac{1}{2}}}{h_1}=0, \tag{8}$$

$$\frac{\theta_{i,j}^{n+1}-\theta_{i,j}^{n+\frac{1}{2}}}{\tau}=\Lambda_2\left(\theta_{i,j}^{n+1}-\theta_{i,j}^{n}\right)-\varepsilon\left(\theta_{i,j}^{n+1}-\theta_{i,j}^{n}\right), \tag{9}$$

$$j=n_1-1;\ \frac{\theta_{i,j}^{n+1}-\theta_{i,j+1}^{n+1}}{h_2}=0, \tag{10}$$

$$j=0;\ \frac{\theta_{i,j+1}^{n+1}-\theta_{i,j}^{n+1}}{h_2}=-6\left[\frac{i}{m}-\left(\frac{i}{m}\right)^2\right], \tag{11}$$

$$\beta\Lambda\varphi_{i,j}^{n+1}=\theta_{i,j}^{n+1}-\varphi_{i,j}^{n+1}, \tag{12}$$

$$i=0;\ \varphi_{i,j}^{n+1}=0. \tag{13}$$

Here

$$\Lambda_1\theta_{i,j}\sim\frac{\partial^2\theta}{\partial\xi^2};\quad \Lambda_2\theta_{i,j}\sim\frac{\partial^2\theta}{\partial\eta^2};\quad \Lambda\varphi_{i,j}\sim\frac{\partial\varphi}{\partial\xi}\ \left(\Lambda\varphi_{i,j}=\Lambda_{h_1}^{+}\varphi_{i,j}\right).$$

Having eliminated $\theta_{i,j}^{n+1/2}$ from (7) and (9), we find

$$\frac{\theta_{i,j}^{n+1}-\theta_{i,j}^{n}}{\tau}=\Lambda_1\theta_{i,j}^{n+1}+\Lambda_2\theta_{i,j}^{n+1}-\varepsilon\left(\theta_{i,j}^{n+1}-\varphi_{i,j}^{n}\right)-\tau\Lambda_1\left[\Lambda_2\left(\theta_{i,j}^{n+1}-\theta_{i,j}^{n}\right)-\varepsilon\left(\theta_{i,j}^{n+1}-\theta_{i,j}^{n}\right)\right]. \tag{14}$$

The approximation of differential equations (1) and (5) follows from (12) and (14).

Harmonic analysis of the stability of the difference scheme yields an expression for the coefficient of the error increase, i.e.,

$$\rho=\frac{(1+\tau^2a_1a_2+\tau^2\varepsilon a_1)(1+\beta a)+\tau\varepsilon}{(1+\tau a_1+\tau a_2+\tau\varepsilon+\tau^2a_1a_2+\tau^2\varepsilon a_1)(1+\beta a)}, \tag{15}$$

where

$$a_1=\frac{4}{h_1^2}\sin^2\frac{kh_1}{2};\quad a_2=\frac{4}{h_2^2}\sin^2\frac{kh_2}{2};\quad a=\frac{\exp(ikh_1)-1}{h_1}.$$

Here $i=\sqrt{-1}$.

It follows from (15) that $\rho<1$ for all k.

This examination of the stability of the difference scheme also shows that the error does not increase with each subsequent interval, even when the differential operator $d\varphi/d\xi$ is approximated by the left-hand or central difference ratio.

Thus the adopted difference scheme for the integration of problem (1)-(6) exhibits absolute approximation and stability.

Scheme (7)-(13) is realized by means of a scalar run-through for each fractional interval.

In practical calculations for heat transfer, we have to find the magnitude of the temperature for the most heated point on the surface of the body being cooled, and its location along the length of the surface.

Using the difference scheme (7)-(13), we determined the temperatures θ_{max} on a digital computer for the hottest spot on the base of the fin or for the porous strip being cooled, and we did this for various values of ε, β, and B_0. The error in the calculation with this scheme is no more than 2.5-3%.

The values of θ_{max} are shown in Fig. 1a-d, and here we also find the curves for $\xi/B_0=f(\varepsilon,\beta,B_0)$, enabling us to determine the location of the hottest point on the base of the fin or on the porous strip, relative to the entry of the cooling fluid into the channels.

The average fluid temperature at the outlet from the channels of the extended surface area of the porous strip being cooled can be determined [5] by means of the formula

$$\varphi_{av} = \frac{B_0}{\varepsilon\beta}. \tag{16}$$

It follows from an analysis of the curves for $\xi/B_0 = f(\varepsilon, \beta, B_0)$ that the hottest point on the surface shifts from the midsection (where we have the maximum heat flow) toward the outlet of the cooling fluid from the channel.

The resulting graphical relationships $\theta_{max} = f(\varepsilon, \beta, B_0)$ permit us to introduce significant correction factors into the solution for the problem relating to the cooling of extended and porous surfaces when the heat flow is distributed parabolically, as opposed to the calculation which would be undertaken on the basis of results from a study of the heat conduction of a finned surface in the case of a constant heat flow [5].

For example, comparing θ_{max} and θ [5] for certain values of ε, β, and B_0, we see that the error in the determination of θ_{max} – on the basis of the calculation material given in [5] – may reach 30%.

NOTATION

t and u	are the temperatures of the fin (of the porous material) and of the fluid, respectively;
q_{av}	is the average intensity of the heat flow over the length of the surface;
h and B	are the height and the length of the fin or of the porous strip being cooled;
δ	is the thickness of the fin;
α	is the heat-transfer coefficient for the surface of the fin or of the pores containing the medium;
$\varkappa$	is the width of the channel between the fins;
c and γ	are, respectively, the heat capacity and the specific weight of the fluid;
v	is the average fluid velocity over the width of the channel;
x and y	are the coordinate axes (x is directed along the fin or along the porous strip in the direction of fluid motion, and y is directed along the height of the fin or the porous material);
θ and φ	are the dimensionless temperatures of the material and the fluid, respectively;
ξ and η	are dimensionless coordinates;
λ	is the coefficient of thermal conductivity for the material of the fin;
λ_{eq1} and λ_{eq2}	are the equivalent coefficients of thermal conductivity for the porous material, respectively, in the direction of the x- and y-axes;
p	is the pore surface per unit volume of the strip being cooled;
G	is the weight flow rate of the cooling fluid per unit cross-sectional area of the porous surface;
τ	is the iteration step;
n	is the number of the iteration step;
i and j	are the step indices for the ξ- and η-axes, respectively;
h_1 and h_2	are the spacings of the coordinate grid, respectively, in the ξ- and η-directions;
m and n_1	are the number of intervals in the coordinate grid, respectively, for the ξ- and η-axes;
$\Lambda_1\theta$, $\Lambda_2\theta$, $\Lambda\varphi$, a_1, a_2, and a	are the difference operators and the corresponding values of the eigenfunctions.

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